

# Topic 6

## Tables

# 2001 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

$t$ (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function  $W$  of time  $t$ . The table above shows the water temperature as recorded every 3 days over a 15-day period.
- Use data from the table to find an approximation for  $W'(12)$ . Show the computations that lead to your answer. Indicate units of measure.
  - Approximate the average temperature, in degrees Celsius, of the water over the time interval  $0 \leq t \leq 15$  days by using a trapezoidal approximation with subintervals of length  $\Delta t = 3$  days.
  - A student proposes the function  $P$ , given by  $P(t) = 20 + 10te^{(-t/3)}$ , as a model for the temperature of the water in the pond at time  $t$ , where  $t$  is measured in days and  $P(t)$  is measured in degrees Celsius. Find  $P'(12)$ . Using appropriate units, explain the meaning of your answer in terms of water temperature.
  - Use the function  $P$  defined in part (c) to find the average value, in degrees Celsius, of  $P(t)$  over the time interval  $0 \leq t \leq 15$  days.
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# 2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

$x$	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let  $f$  be a function that is differentiable for all real numbers. The table above gives the values of  $f$  and its derivative  $f'$  for selected points  $x$  in the closed interval  $-1.5 \leq x \leq 1.5$ . The second derivative of  $f$  has the property that  $f''(x) > 0$  for  $-1.5 \leq x \leq 1.5$ .
- (a) Evaluate  $\int_0^{1.5} (3f'(x) + 4) dx$ . Show the work that leads to your answer.
- (b) Write an equation of the line tangent to the graph of  $f$  at the point where  $x = 1$ . Use this line to approximate the value of  $f(1.2)$ . Is this approximation greater than or less than the actual value of  $f(1.2)$ ? Give a reason for your answer.
- (c) Find a positive real number  $r$  having the property that there must exist a value  $c$  with  $0 < c < 0.5$  and  $f''(c) = r$ . Give a reason for your answer.
- (d) Let  $g$  be the function given by  $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \geq 0 \end{cases}$ .
- The graph of  $g$  passes through each of the points  $(x, f(x))$  given in the table above. Is it possible that  $f$  and  $g$  are the same function? Give a reason for your answer.

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**END OF EXAMINATION**

**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

Distance $x$ (cm)	0	1	5	6	8
Temperature $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature  $T(x)$ , in degrees Celsius ( $^{\circ}\text{C}$ ), of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.
- (a) Estimate  $T'(7)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of  $T(x)$  for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find  $\int_0^8 T'(x) dx$ , and indicate units of measure. Explain the meaning of  $\int_0^8 T'(x) dx$  in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that  $T''(x) > 0$  for every  $x$  in the interval  $0 < x < 8$ ? Explain your answer.
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**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF PART A OF SECTION II**

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part B****Time—45 minutes  
Number of problems—3****No calculator is allowed for these problems.**

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$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

4. Rocket A has positive velocity  $v(t)$  after being launched upward from an initial height of 0 feet at time  $t = 0$  seconds. The velocity of the rocket is recorded for selected values of  $t$  over the interval  $0 \leq t \leq 80$  seconds, as shown in the table above.
- (a) Find the average acceleration of rocket A over the time interval  $0 \leq t \leq 80$  seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of  $\int_{10}^{70} v(t) dt$  in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate  $\int_{10}^{70} v(t) dt$ .
- (c) Rocket B is launched upward with an acceleration of  $a(t) = \frac{3}{\sqrt{t+1}}$  feet per second per second. At time  $t = 0$  seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time  $t = 80$  seconds? Explain your answer.
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .
- (a) Explain why there must be a value  $r$  for  $1 < r < 3$  such that  $h(r) = -5$ .
- (b) Explain why there must be a value  $c$  for  $1 < c < 3$  such that  $h'(c) = -5$ .
- (c) Let  $w$  be the function given by  $w(x) = \int_1^{g(x)} f(t) dt$ . Find the value of  $w'(3)$ .
- (d) If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF PART A OF SECTION II**

$t$ (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function  $r$  of time  $t$ , where  $t$  is measured in minutes. For  $0 < t < 12$ , the graph of  $r$  is concave down. The table above gives selected values of the rate of change,  $r'(t)$ , of the radius of the balloon over the time interval  $0 \leq t \leq 12$ . The radius of the balloon is 30 feet when  $t = 5$ .

(Note: The volume of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .)

- (a) Estimate the radius of the balloon when  $t = 5.4$  using the tangent line approximation at  $t = 5$ . Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when  $t = 5$ . Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate  $\int_0^{12} r'(t) dt$ . Using correct units, explain the meaning of  $\int_0^{12} r'(t) dt$  in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than  $\int_0^{12} r'(t) dt$ ? Give a reason for your answer.

2007

2009 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

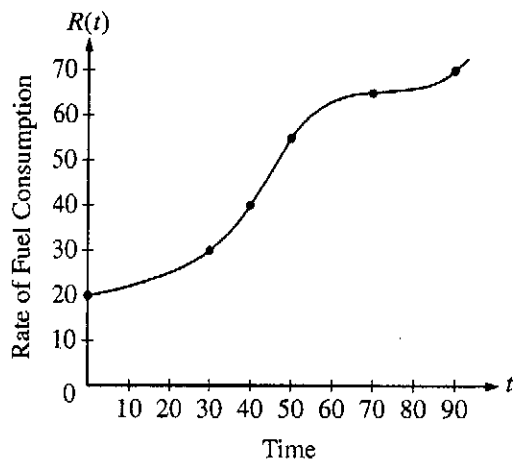
$x$	2	3	5	8	13
$f(x)$	1	4	-2	3	6

5. Let  $f$  be a function that is twice differentiable for all real numbers. The table above gives values of  $f$  for selected points in the closed interval  $2 \leq x \leq 13$ .
- (a) Estimate  $f'(4)$ . Show the work that leads to your answer.
- (b) Evaluate  $\int_2^{13} (3 - 5f'(x)) \, dx$ . Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate  $\int_2^{13} f(x) \, dx$ .  
Show the work that leads to your answer.
- (d) Suppose  $f'(5) = 3$  and  $f''(x) < 0$  for all  $x$  in the closed interval  $5 \leq x \leq 8$ . Use the line tangent to the graph of  $f$  at  $x = 5$  to show that  $f(7) \leq 4$ . Use the secant line for the graph of  $f$  on  $5 \leq x \leq 8$  to show that  $f(7) \geq \frac{4}{3}$ .

WRITE ALL WORK IN THE PINK EXAM BOOKLET.



2003 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.
- Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
  - The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
  - Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
  - For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane.  
Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

# 2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

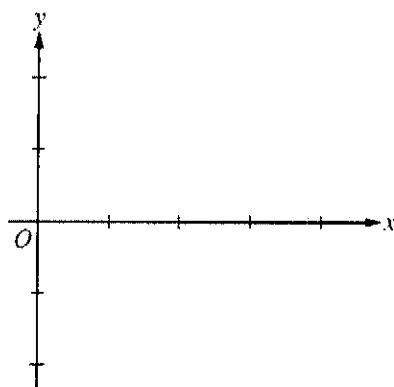
## CALCULUS AB SECTION II, Part B

Time—45 minutes  
Number of problems—3

No calculator is allowed for these problems.

$x$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let  $f$  be a function that is continuous on the interval  $[0, 4]$ . The function  $f$  is twice differentiable except at  $x = 2$ . The function  $f$  and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of  $f$  do not exist at  $x = 2$ .
- (a) For  $0 < x < 4$ , find all values of  $x$  at which  $f$  has a relative extremum. Determine whether  $f$  has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (b) On the axes provided, sketch the graph of a function that has all the characteristics of  $f$ .  
(Note: Use the axes provided in the pink test booklet.)



- (c) Let  $g$  be the function defined by  $g(x) = \int_1^x f(t) dt$  on the open interval  $(0, 4)$ . For  $0 < x < 4$ , find all values of  $x$  at which  $g$  has a relative extremum. Determine whether  $g$  has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (d) For the function  $g$  defined in part (c), find all values of  $x$ , for  $0 < x < 4$ , at which the graph of  $g$  has a point of inflection. Justify your answer.

WRITE ALL WORK IN THE TEST BOOKLET.

**2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**